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MODELING OF ASCENDING POLYDISPERSE FLOW WITH ALLOWANCE FOR HETEROGENEOUS COMBUSTION, PARTICLE ROTATION, AND TURBULENT AND PSEUDOTURBULENT EFFECTS

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A closed system of equations that describes motion and physicochemical processes in ascending polydisperse flow with allowance for the rotation of the dispersed phase, the aerodynamic-drag, gravity, and Magnus forces, and the interaction of particles with each other and with the reactor wall is constructed. The equations of motion and heat transfer of the solid phase are closed at the level of equations for the second moments of pulsations of the linear and angular velocities of particles and their temperature. The pulsation characteristics of the carrying medium are computed using the equation for the turbulent gas energy.

Keywords: *combustion, particle, gas, polydispersity, turbulence, pseudoturbulence, temperature, pulsations, heterogeneous reaction, concentration, velocity, moments, aerodynamics, rotation.*

Introduction. Considerable attention is given to the mathematical modeling of the aerodynamics, heat and mass exchange, and combustion of a polydisperse ensemble of coke-ash particles with the aim of obtaining detailed information on the distribution of the velocity, concentration, and temperature fields of phases; this information is necessary for designing power-generating plants. Models including the equations of transfer of second moments of pulsations of the particle velocity and temperature have gained the greatest acceptance. However, the effects associated with interparticle interaction (pseudoturbulent effects), polydispersity, the rotation of the dispersed phase, and the action of the Magnus force are not allowed for in full measure in these equations. In [1, 2], the equations of momentum of the solid phase are closed using the equations for the second moments of pulsations of the translational velocity of monodisperse particles with allowance for the turbulent and pseudoturbulent mechanisms of transfer. A model of gas-suspension flow with the equations for the correlation moments of fluctuations of the linear and angular velocities of monodisperse particles with allowance for turbulent effects and for the action of the Magnus force is given in [3]. Extra terms of nonturbulent origin that are associated with the pseudoturbulent mechanism of transfer of momentum and moment of momentum appear in the above equations in [4]. It is shown that allowance for the rotation of particles contributes to a more accurate description of the dependence $\langle u'^2_{pi} \rangle^{0.5}(r)$ compared to the models that disregard this effect. In [5], the equations of momentum and heat transfer in the solid phase are closed on the basis of equations for correlations of second order of fluctuations of the linear velocity and temperature of particles with allowance for the generation and dissipation of the kinetic energy of random motion on mono- and polydisperse particles due to their collisions and of pulsatory motion of the dispersed phase of turbulent origin by the action of aerodynamic forces.

In the present work, we propose a mathematical model for calculation of aerodynamics and thermal and physicochemical processes within the framework of the so-called two-fluid models (Eulerian approximation); this model has been constructed with the equations of transfer of second moments of pulsations of the particle (linear and angular) velocity and temperature with allowance for radiant and convective heat exchange, heterogeneous reactions, interphase (aerodynamic-drag and Magnus) and interparticle interaction forces, turbulent and pseudoturbulent effects, polydispersity, and particle rotation.

In constructing the computational procedure, we make the following assumptions: 1) the process is stationary; 2) the stoichiometric reaction scheme includes the heterogeneous reaction $C + O_2 = CO_2$ that occurs on the exterior surface of impermeable spherical particles; 3) change in the gas pressure in the cross section of the flow is disre-

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garded; 4) consideration is given to a two-phase heterogeneous medium consisting of the carrying medium (nitrogen, oxygen, carbon dioxide) and the solid phase presented in the form of two ensembles (coal + ash), each of a finite number of monodisperse fractions M_C and M_{ash}; 5) the vector of angular particle velocity is directed along the transverse axis; 6) the boundary-layer approximation, $v << u$, $u' << u$, $v' \sim v$, and $\partial/\partial z << \partial/\partial r$, is used.

Basic Equations. The system of equations that describes the motion, heat and mass exchange, and chemical reaction of the ascending polydisperse flow of coke-ash particles in an axisymmetric channel has the form

$$\frac{\partial (C_{O_2} u_g)}{\partial z} + \frac{\partial [r (C_{O_2} v_g + \langle C'_{O_2} v'_g \rangle)]}{r \partial r} = -6 \sum_{j=1}^{M_C} \frac{L_j S_j \beta_j C_{O_2}}{(L_j + S_j)\, \delta_j} , \quad (1)$$

$$\frac{\partial (C_{CO_2} u_g)}{\partial z} + \frac{\partial [r (C_{CO_2} v_g + \langle C'_{CO_2} v'_g \rangle)]}{r \partial r} = 6 \sum_{j=1}^{M_C} \frac{L_j S_j \beta_j C_{O_2}}{(L_j + S_j)\, \delta_j} , \quad (2)$$

$$\frac{\partial (C_{N_2} u_g)}{\partial z} + \frac{\partial [r (C_{N_2} v_g + \langle C'_{N_2} v'_g \rangle)]}{r \partial r} = 0 , \quad (3)$$

$$\frac{\partial (\rho_g u_g)}{\partial z} + \frac{\partial [r (\rho_g v_g + \langle \rho'_g v'_g \rangle)]}{r \partial r} = 6 \sum_{j=1}^{M_C} \frac{(\mu_{CO_2} - \mu_{O_2})\, L_j S_j \beta_j C_{O_2}}{(L_j + S_j)\, \delta_j} , \quad (4)$$

$$\frac{\partial (\beta_j u_{pj})}{\partial z} + \frac{\partial [r (\beta_j v_{pj} + \langle \beta'_j v'_{pj} \rangle)]}{r \partial r} = -\frac{6 \mu_C L_j S_j \beta_j C_{O_2}}{(L_j + S_j)\, \delta_j \rho_C} , \quad (5)$$

$$\frac{\partial (n_j u_{pj})}{\partial z} + \frac{\partial [r (n_j v_{pj} + \langle n'_j v'_{pj} \rangle)]}{r \partial r} = 0 , \quad (6)$$

$$\frac{\partial (\beta_l u_{pl})}{\partial z} + \frac{\partial [r (\beta_l v_{pl} + \langle \beta'_l v'_{pl} \rangle)]}{r \partial r} = 0 , \quad (7)$$

$$\rho_g u_g \frac{\partial u_g}{\partial z} + (\rho_g v_g + \langle \rho'_g v'_g \rangle) \frac{\partial u_g}{\partial r} = \frac{\partial}{r \partial r} \left[r \rho_g \left(\eta_g \frac{\partial u_g}{\partial r} - \langle u'_g v'_g \rangle \right) \right] - \frac{\partial P}{\partial z} - \sum_{i=1}^{M_C + M_{ash}} (F_{aiz} + F_{Miz})$$
$$+ 6 \sum_{j=1}^{M_C} \frac{(\mu_{CO_2} - \mu_{O_2})\, L_j S_j \beta_j C_{O_2}}{(L_j + S_j)\, \delta_j} (u_{pj} - u_g) , \quad (8)$$

$$\rho_{pi} \left[\beta_i u_{pi} \frac{\partial u_{pi}}{\partial z} + (\beta_i v_{pi} + \langle \beta'_i v'_{pi} \rangle) \frac{\partial u_{pi}}{\partial r} \right] = -\frac{\rho_{pi}}{r} \frac{\partial (\beta_i r \langle u'_{pi} v'_{pi} \rangle)}{\partial r} + F_{aiz} + F_{Miz} + F_{coliz} - \rho_{pi} \beta_i g , \quad (9)$$

$$\rho_{pi}\left[\beta_i u_{pi}\frac{\partial v_{pi}}{\partial z}+(\beta_i v_{pi}+\langle\beta_i' v_{pi}'\rangle)\frac{\partial v_{pi}}{\partial r}\right]=-\frac{\rho_{pi}}{r}\frac{\partial\,[r\,(v_{pi}\,\langle\beta_i' v_{pi}'\rangle+\beta_i\,\langle v_{pi}'^{2}\rangle)]}{\partial r}$$

$$+F_{air}+F_{Mir}+F_{colir}+\frac{\rho_{pi}\beta_i\,\langle w_{pi}'^{2}\rangle}{r}\,, \tag{10}$$

$$\beta_i u_{pi}\frac{\partial\omega_{\varphi i}}{\partial z}+(\beta_i v_{pi}+\langle\beta_i' v_{pi}'\rangle)\frac{\partial\omega_{\varphi i}}{\partial r}=-\frac{\partial\,(r\beta_i\,\langle\omega_{\varphi i}' v_{pi}'\rangle)}{r\partial r}-\frac{\beta_i\,\langle\omega_{ri}' w_{pi}'\rangle}{r}-\gamma_i\beta_i\left(\omega_{\varphi i}+\frac{\partial u_g}{2\partial r}\right), \tag{11}$$

$$\rho_g u_g\frac{\partial k_g}{\partial z}+(\rho_g v_g+\langle\rho_g' v_g'\rangle)\frac{\partial k_g}{\partial r}=\frac{\partial}{r\partial r}\left[r\rho_g\left(\frac{\eta_{t,g}}{\sigma_k}+\eta_g\right)\frac{\partial k_g}{\partial r}\right]+\rho_g\eta_{t,g}\left(\frac{\partial u_g}{\partial r}\right)^2$$

$$-\rho_g\,(\varepsilon_g+\sum_{i=1}^{M_C+M_{ash}}\varepsilon_{pi})+\sum_{i=1}^{M_C+M_{ash}}G_{gi}-6\sum_{j=1}^{M_C}\frac{(\mu_{CO_2}-\mu_{O_2})\,L_jS_j\beta_jC_{O_2}}{(L_j+S_j)\,\delta_j}\,(2k_g-\langle u_{pj}'u_g'\rangle-\langle v_{pj}'v_g'\rangle-\langle w_{pj}'w_g'\rangle)\,, \tag{12}$$

$$c_g\rho_g u_g\frac{\partial t_g}{\partial z}+c_g\,(\rho_g v_g+\langle\rho_g' v_g'\rangle)\frac{\partial t_g}{\partial r}=\frac{\partial}{r\partial r}\left[rc_g\rho_g\left(\frac{\eta_g}{\mathrm{Pr}_g}\frac{\partial t_g}{\partial r}-\langle t_g' v_g'\rangle\right)\right]+10^{-3}\,u_g\frac{\partial P}{\partial z}$$

$$+10^{-3}\sum_{i=1}^{M_C+M_{ash}}(F_{aiz}+F_{Miz})\,(u_g-u_{pi})+\sum_{i=1}^{M_C+M_{ash}}\alpha_{\Sigma i}\,(t_{pi}-t_g)\frac{6\beta_i}{\delta_i}+6\sum_{j=1}^{M_C}\frac{(\mu_{CO_2}-\mu_{O_2})\,L_jS_j\beta_jC_{O_2}}{(L_j+S_j)\,\delta_j}$$

$$\times\left[c_{pj}t_{pj}-c_gt_g+\frac{0.5\,(u_g-u_{pj})^2}{10^3}\right]+10^{-3}\,\rho_g\eta_{t,g}\left(\frac{\partial u_g}{\partial r}\right)^2, \tag{13}$$

$$c_{pi}\beta_i u_{pi}\frac{\partial t_{pi}}{\partial z}+c_{pi}\,(\beta_i v_{pi}+\langle\beta_i' v_{pi}'\rangle)\frac{\partial t_{pi}}{\partial r}=-\frac{\partial\,(r\beta_i c_{pi}\,\langle t_{pi}' v_{pi}'\rangle)}{r\partial r}-\alpha_{\Sigma i}\,(t_{pi}-t_g)\frac{6\beta_i}{\delta_i\rho_{pi}}$$

$$+\frac{6\theta_iL_iS_i\beta_iC_{O_2}Q_i}{(L_i+S_i)\,\delta_i\rho_{pi}}\,, \tag{14}$$

$$\frac{dB_g}{dz}=12\pi\int_0^R\sum_{j=1}^{M_C}\frac{(\mu_{CO_2}-\mu_{O_2})\,L_jS_j\beta_jC_{O_2}}{(L_j+S_j)\,\delta_j}\,rdr\,, \tag{15}$$

where we have

$$F_{Mir}=-\lambda_{\omega i}\beta_i\rho_{pi}\,(u_g-u_{pi})\left(\omega_{\varphi i}+\frac{\partial u_g}{2\partial r}\right);\quad\langle t_g' v_g'\rangle=-\frac{\eta_{t,g}}{\mathrm{Pr}_{t,g}}\frac{\partial t_g}{\partial r}\,,\quad\langle n_j' v_{pj}'\rangle=-J_{pj}\frac{\partial n_j}{\partial r}\,;$$

$$F_{Miz}=\lambda_{\omega i}\beta_i\rho_{pi}\left[\langle\omega_{ri}' w_{pi}'\rangle-\langle\omega_{\varphi i}' v_{pi}'\rangle+(v_g-v_{pi})\left(\omega_{\varphi i}+\frac{\partial u_g}{2\partial r}\right)\right];\quad\langle C_\chi' v_g'\rangle=-J_g\frac{\partial C_\chi}{\partial r}\,;$$

$$\langle \beta_i' v_{pi}' \rangle = -J_{pi}\frac{\partial \beta_i}{\partial r}\,;\quad \langle u_{pi}' v_{pi}' \rangle = -\eta_{t,pi}\frac{\partial u_{pi}}{\partial r}\,;\quad \lambda_{\omega i} = \frac{3\rho_g}{4\rho_{pi}}\,;\quad \langle \rho_g' v_g' \rangle = -J_g\frac{\partial \rho_g}{\partial r}\,;$$

$$\langle u_g' v_g' \rangle = -\eta_{t.g}\frac{\partial u_g}{\partial r}\,;\quad \gamma_i = \frac{60\rho_g\eta_g}{\rho_{pi}\delta_i^2}\,;\quad F_{air} = \frac{\rho_{pi}\beta_i}{\tau_i}(v_g - v_{pi})\,;\quad F_{aiz} = \frac{\rho_{pi}\beta_i}{\tau_i}(u_g - u_{pi})\,;$$

$$\rho_g = \frac{10^{-3}P}{(t_g + 273)\,H\sum_{\chi=1}^{3}\frac{Z_\chi}{\mu_\chi}}\,;\quad \delta_j = \sqrt[3]{\frac{6\beta_j}{\pi n_j}}\,. \qquad (16)$$

Here the subscripts $j = 1 - M_C$ refer to the coke particles, $l = 1 - m_{ash}$ refer to the ash particles; $\chi = 1 - 3 - O_2$, CO_2, N_2; $i = j$ and l; $\theta_j = 1$ and $\theta_l = 0$. The double correlations of pulsations of the gas and particle velocities $\langle u_{pj}' u_g' \rangle$, $\langle v_{pj}' v_g' \rangle$, and $\langle w_{pj}' w_g' \rangle$, which are present in (12), are determined according to [6]. The right-hand sides of the equations of continuity of phases (1), (2), (4), and (5) allow for the influence of the heterogeneous reaction $C + O_2 = CO_2$. The viscous and Reynolds stresses, the pressure gradient, and the aerodynamic-drag and Magnus forces appear in the equation of motion of the gas. The last term of the equation allows for the passage of the burnt carbon into the gas phase (i.e., its acceleration from the j-particle velocity to the gas velocity). Equations (9) and (10) involve the turbulent stresses, the forces of gravity and of interphase and interparticle F_{coli} [7] interaction, and the centrifugal force due to the transverse pulsations of the particle velocity. On the right-hand side of (12), the first term describes the molecular and turbulent transfer of pulsation energy, the second term describes its generation due to the averaged-motion energy, the third term describes its dissipation caused by the viscosity of the gas phase and of the particles present in it, the fourth term describes the generation of turbulent energy in the wakes of the particles, and the last term describes the expenditure of the pulsation gas energy that is associated with a new substance. The molecular and turbulent transfer of the gas flow, the work of Reynolds stresses and of pressure and interphase-interaction forces, the radiant and convective heat exchange between the gas and the particles, and the excess of enthalpy and the deficiency of kinetic energy of the part of the coke-particle substance that becomes a gas due to the heterogeneous reaction, are allowed for in (13). Terms allowing for the pulsation heat transfer in the solid phase, heat exchange between the carrying medium and the dispersed phase, and heat release due to the heterogeneous chemical reaction appear in (14).

To close the system of equations (1)–(16) we must determine the unknown second moments $\langle v_{pi}'^2 \rangle$, $\langle w_{pi}'^2 \rangle$, $\langle t_{pi}' v_{pi}' \rangle$, $\langle \omega_{\varphi i}' v_{pi}' \rangle$, and $\langle \omega_{ri}' w_{pi}' \rangle$ appearing in Eqs. (10), (11), and (14), which in turn are dependent on the Reynolds stresses $\langle w_{pi}' v_{pi}' \rangle$, $\langle u_{pi}'^2 \rangle$, $\langle t_{pi}' w_{pi}' \rangle$, $\langle t_{pi}' \omega_{ri}' \rangle$, $\langle t_{pi}' \omega_{\varphi i}' \rangle$, $\langle \omega_{\varphi i}' \omega_{ri}' \rangle$, $\langle \omega_{\varphi i}'^2 \rangle$, $\langle \omega_{ri}'^2 \rangle$, $\langle \omega_{ri}' v_{pi}' \rangle$, $\langle \omega_{\varphi i}' w_{pi}' \rangle$, and $\langle u_{pi}' \omega_{\varphi i}' \rangle$ (see below). For computation of the indicated variables, we use a special computational procedure that is based on construction of the equations of transfer of the sought correlations [8–10]. From this procedure, in the approximation of a narrow channel, we obtain a closed description of the motion and heat transfer of the dispersed phase at the level of equations for the second moments of pulsations of the particle (translational and angular) velocity and temperature

$$\beta_i u_{pi}\frac{\partial\langle \omega_{ri}' w_{pi}' \rangle}{\partial z} + (\beta_i v_{pi} + \langle \beta_i' v_{pi}' \rangle)\frac{\partial\langle \omega_{ri}' w_{pi}' \rangle}{\partial r} = \frac{\partial}{r\partial r}\left(\frac{r\beta_i\langle v_{pi}'^2 \rangle\,\partial\langle \omega_{ri}' w_{pi}' \rangle}{\psi_{1i}\partial r}\right)$$

$$+ \frac{\partial}{r\partial r}\left(\frac{r\beta_i\langle w_{pi}' v_{pi}' \rangle\,\partial\langle \omega_{ri}' v_{pi}' \rangle}{\psi_{1i}\partial r}\right) + \frac{\partial}{r\partial r}\left(\frac{r\beta_i\langle \omega_{ri}' v_{pi}' \rangle\,\partial\langle w_{pi}' v_{pi}' \rangle}{\psi_{1i}\partial r}\right) + \frac{\beta_i\omega_{\varphi i}\langle w_{pi}'^2 \rangle}{r}$$

$$- \frac{\beta_i\langle w_{pi}' v_{pi}' \rangle\,\partial\langle \omega_{\varphi i}' w_{pi}' \rangle}{r\psi_{2i}\partial r} - \frac{\beta_i\langle \omega_{\varphi i}' v_{pi}' \rangle\,\partial\langle w_{pi}'^2 \rangle}{2r\psi_{2i}\partial r} - \frac{\beta_i v_{pi}\langle \omega_{ri}' w_{pi}' \rangle}{r}$$

$$+ \frac{\beta_i\langle v_{pi}'^2 \rangle\,\partial\langle \omega_{ri}' w_{pi}' \rangle}{r\psi_{1i}\partial r} + \frac{\beta_i\langle w_{pi}' v_{pi}' \rangle\,\partial\langle \omega_{ri}' v_{pi}' \rangle}{r\psi_{1i}\partial r} + \frac{\beta_i\langle \omega_{ri}' v_{pi}' \rangle\,\partial\langle w_{pi}' v_{pi}' \rangle}{r\psi_{1i}\partial r}$$

$$- \beta_i \left(\frac{1}{\tau_i} + \gamma_i\right) \langle \omega'_{ri} w'_{pi} \rangle + \lambda_{\omega i} \beta_i (u_g - u_{pi}) \langle \omega'^2_{ri} \rangle + (G_{p1i} - D_{1i}) , \tag{17}$$

$$2\beta_i u_{pi} \frac{\partial \langle \omega'^2_{ri} \rangle}{\partial z} + 2 (\beta_i v_{pi} + \langle \beta'_i v'_{pi} \rangle) \frac{\partial \langle \omega'^2_{ri} \rangle}{\partial r} = \frac{\partial}{r \partial r} \left(\frac{r \beta_i \langle v'^2_{pi} \rangle \partial \langle \omega'^2_{ri} \rangle}{\psi_{3i} \partial r} \right)$$

$$+ \frac{2\partial}{r\partial r} \left(\frac{r\beta_i \langle \omega'_{ri} v'_{pi} \rangle \partial \langle \omega'_{ri} v'_{pi} \rangle}{\psi_{3i} \partial r} \right) + \frac{4\beta_i \omega_{\varphi i} \langle \omega'_{ri} w'_{pi} \rangle}{r} - \frac{4\beta_i \langle w'_{pi} v'_{pi} \rangle \partial \langle \omega'_{\varphi i} \omega'_{ri} \rangle}{r \psi_{4i} \partial r}$$

$$- \frac{4\beta_i \langle \omega'_{\varphi i} v'_{pi} \rangle \partial \langle \omega'_{ri} w'_{pi} \rangle}{r \psi_{4i} \partial r} - \frac{4\beta_i \langle \omega'_{ri} v'_{pi} \rangle \partial \langle \omega'_{\varphi i} w'_{pi} \rangle}{r \psi_{4i} \partial r} - 4\gamma_i \beta_i \langle \omega'^2_{ri} \rangle + 4 (G_{p3i} - D_{3i}) , \tag{18}$$

$$2\beta_i u_{pi} \frac{\partial \langle \omega'^2_{\varphi i} \rangle}{\partial z} + 2 (\beta_i v_{pi} + \langle \beta'_i v'_{pi} \rangle) \frac{\partial \langle \omega'^2_{\varphi i} \rangle}{\partial r} = \frac{\partial}{r \partial r} \left(\frac{r \beta_i \langle v'^2_{pi} \rangle \partial \langle \omega'^2_{\varphi i} \rangle}{\psi_{3i} \partial r} \right)$$

$$+ \frac{2\partial}{r\partial r} \left(\frac{r\beta_i \langle \omega'_{\varphi i} v'_{pi} \rangle \partial \langle \omega'_{\varphi i} v'_{pi} \rangle}{\psi_{3i} \partial r} \right) - \frac{4\beta_i \langle \omega'_{\varphi i} v'_{pi} \rangle \partial \omega_{\varphi i}}{\partial r} + \frac{4\beta_i \langle w'_{pi} v'_{pi} \rangle \partial \langle \omega'_{\varphi i} \omega'_{ri} \rangle}{r \psi_{4i} \partial r}$$

$$+ \frac{4\beta_i \langle \omega'_{\varphi i} v'_{pi} \rangle \partial \langle \omega'_{ri} w'_{pi} \rangle}{r \psi_{4i} \partial r} + \frac{4\beta_i \langle \omega'_{ri} v'_{pi} \rangle \partial \langle \omega'_{\varphi i} w'_{pi} \rangle}{r \psi_{4i} \partial r} - 4\beta_i \gamma_i \langle \omega'^2_{\varphi i} \rangle + 4 (G_{p4i} - D_{4i}) , \tag{19}$$

$$\beta_i u_{pi} \frac{\partial \langle \omega'_{\varphi i} v'_{pi} \rangle}{\partial z} + (\beta_i v_{pi} + \langle \beta'_i v'_{pi} \rangle) \frac{\partial \langle \omega'_{\varphi i} v'_{pi} \rangle}{\partial r} = \frac{\partial}{r \partial r} \left(\frac{r \beta_i \langle v'^2_{pi} \rangle \partial \langle \omega'_{\varphi i} v'_{pi} \rangle}{\psi_{2i} \partial r} \right)$$

$$+ \frac{\partial}{2r\partial r} \left(\frac{r\beta_i \langle \omega'_{\varphi i} v'_{pi} \rangle \partial \langle v'^2_{pi} \rangle}{\psi_{2i} \partial r} \right) - \frac{\beta_i \langle v'^2_{pi} \rangle \partial \omega_{\varphi i}}{\partial r} + \frac{\beta_i \langle v'^2_{pi} \rangle \partial \langle \omega'_{ri} w'_{pi} \rangle}{r \psi_{1i} \partial r}$$

$$+ \frac{\beta_i \langle \omega'_{ri} v'_{pi} \rangle \partial \langle w'_{pi} v'_{pi} \rangle}{r \psi_{1i} \partial r} + \frac{\beta_i \langle w'_{pi} v'_{pi} \rangle \partial \langle \omega'_{ri} v'_{pi} \rangle}{r \psi_{1i} \partial r} - \frac{\beta_i \langle \omega'_{\varphi i} v'_{pi} \rangle \partial v_{pi}}{\partial r}$$

$$- \frac{\beta_i \langle w'_{pi} v'_{pi} \rangle \partial \langle \omega'_{\varphi i} w'_{pi} \rangle}{r \psi_{2i} \partial r} - \frac{\beta_i \langle \omega'_{\varphi i} v'_{pi} \rangle \partial \langle w'^2_{pi} \rangle}{2r \psi_{2i} \partial r} - \beta_i \left(\frac{1}{\tau_i} + \gamma_i\right) \langle \omega'_{\varphi i} v'_{pi} \rangle$$

$$- \lambda_{\omega i} \beta_i (u_g - u_{pi}) \langle \omega'^2_{\varphi i} \rangle + (G_{p2i} - D_{2i}) , \tag{20}$$

$$\beta_i u_{pi} \frac{\partial \langle \omega'_{\varphi i} \omega'_{ri} \rangle}{\partial z} + (\beta_i v_{pi} + \langle \beta'_i v'_{pi} \rangle) \frac{\partial \langle \omega'_{\varphi i} \omega'_{ri} \rangle}{\partial r} = \frac{\partial}{r \partial r} \left(\frac{r \beta_i \langle v'^2_{pi} \rangle \partial \langle \omega'_{\varphi i} \omega'_{ri} \rangle}{\psi_{4i} \partial r} \right)$$

$$
+\frac{\partial}{r\partial r}\left(\frac{r\beta_i \langle \omega'_{\varphi i} v'_{pi}\rangle\, \partial \langle \omega'_{ri} v'_{pi}\rangle}{\psi_{4i}\partial r}\right)+\frac{\partial}{r\partial r}\left(\frac{r\beta_i \langle \omega'_{ri} v'_{pi}\rangle\, \partial \langle \omega'_{\varphi i} v'_{pi}\rangle}{\psi_{4i}\partial r}\right)-\frac{\beta_i \langle \omega'_{ri} v'_{pi}\rangle\, \partial \omega_{\varphi i}}{\partial r}
$$

$$
+\frac{\beta_i \langle w'_{pi} v'_{pi}\rangle\, \partial \langle \omega'^{2}_{ri}\rangle}{2r\psi_{3i}\partial r}+\frac{\beta_i \langle \omega'_{ri} v'_{pi}\rangle\, \partial \langle \omega'_{ri} w'_{pi}\rangle}{r\psi_{3i}\partial r}+\frac{\beta_i \omega_{\varphi i} \langle \omega'_{\varphi i} w'_{pi}\rangle}{r}
$$

$$
-\frac{\beta_i \langle w'_{pi} v'_{pi}\rangle\, \partial \langle \omega'^{2}_{\varphi i}\rangle}{2r\psi_{3i}\partial r}-\frac{\beta_i \langle \omega'_{\varphi i} v'_{pi}\rangle\, \partial \langle \omega'_{\varphi i} w'_{pi}\rangle}{r\psi_{3i}\partial r}-2\beta_i\gamma_i \langle \omega'_{\varphi i}\omega'_{ri}\rangle\ , \tag{21}
$$

$$
\beta_i u_{pi}\frac{\partial \langle \omega'_{ri} v'_{pi}\rangle}{\partial z}+(\beta_i v_{pi}+\langle \beta'_i v'_{pi}\rangle)\frac{\partial \langle \omega'_{ri} v'_{pi}\rangle}{\partial r}=\frac{\partial}{r\partial r}\left(\frac{r\beta_i \langle v'^{2}_{pi}\rangle\, \partial \langle \omega'_{ri} v'_{pi}\rangle}{\psi_{2i}\partial r}\right)
$$

$$
+\frac{\partial}{2r\partial r}\left(\frac{r\beta_i \langle \omega'_{ri} v'_{pi}\rangle\, \partial \langle v'^{2}_{pi}\rangle}{\psi_{2i}\partial r}\right)+\frac{\beta_i \omega_{\varphi i} \langle w'_{pi} v'_{pi}\rangle}{r}-\frac{\beta_i \langle v'^{2}_{pi}\rangle\, \partial \langle \omega'_{\varphi i} w'_{pi}\rangle}{r\psi_{5i}\partial r}
$$

$$
-\frac{\beta_i \langle \omega'_{\varphi i} v'_{pi}\rangle\, \partial \langle w'_{pi} v'_{pi}\rangle}{r\psi_{5i}\partial r}-\frac{\beta_i \langle w'_{pi} v'_{pi}\rangle\, \partial \langle \omega'_{\varphi i} v'_{pi}\rangle}{r\psi_{5i}\partial r}-\frac{\beta_i \langle \omega'_{ri} v'_{pi}\rangle\, \partial v_{pi}}{\partial r}
$$

$$
-\frac{\beta_i \langle w'_{pi} v'_{pi}\rangle\, \partial \langle \omega'_{ri} w'_{pi}\rangle}{r\psi_{2i}\partial r}-\frac{\beta_i \langle \omega'_{ri} v'_{pi}\rangle\, \partial \langle w'^{2}_{pi}\rangle}{2r\psi_{2i}\partial r}-\beta_i\left(\frac{1}{\tau_i}+\gamma_i\right)\langle \omega'_{ri} v'_{pi}\rangle
$$

$$
-\lambda_{\omega i}\beta_i (u_g-u_{pi})\langle \omega'_{\varphi i}\omega'_{ri}\rangle\ , \tag{22}
$$

$$
\beta_i u_{pi}\frac{\partial \langle \omega'_{\varphi i} w'_{pi}\rangle}{\partial z}+(\beta_i v_{pi}+\langle \beta'_i v'_{pi}\rangle)\frac{\partial \langle \omega'_{\varphi i} w'_{pi}\rangle}{\partial r}=\frac{\partial}{r\partial r}\left(\frac{r\beta_i \langle v'^{2}_{pi}\rangle\, \partial \langle \omega'_{\varphi i} w'_{pi}\rangle}{\psi_{5i}\partial r}\right)
$$

$$
+\frac{\partial}{r\partial r}\left(\frac{r\beta_i \langle \omega'_{\varphi i} v'_{pi}\rangle\, \partial \langle w'_{pi} v'_{pi}\rangle}{\psi_{5i}\partial r}\right)+\frac{\partial}{r\partial r}\left(\frac{r\beta_i \langle w'_{pi} v'_{pi}\rangle\, \partial \langle \omega'_{\varphi i} v'_{pi}\rangle}{\psi_{5i}\partial r}\right)-\frac{\beta_i \langle w'_{pi} v'_{pi}\rangle\, \partial \omega_{\varphi i}}{\partial r}
$$

$$
+\frac{\beta_i \langle w'_{pi} v'_{pi}\rangle\, \partial \langle \omega'_{ri} w'_{pi}\rangle}{r\psi_{2i}\partial r}+\frac{\beta_i \langle \omega'_{ri} v'_{pi}\rangle\, \partial \langle w'^{2}_{pi}\rangle}{2r\psi_{2i}\partial r}-\frac{\beta_i v_{pi} \langle \omega'_{\varphi i} w'_{pi}\rangle}{r}
$$

$$
+\frac{\beta_i \langle v'^{2}_{pi}\rangle\, \partial \langle \omega'_{\varphi i} w'_{pi}\rangle}{r\psi_{5i}\partial r}+\frac{\beta_i \langle \omega'_{\varphi i} v'_{pi}\rangle\, \partial \langle w'_{pi} v'_{pi}\rangle}{r\psi_{5i}\partial r}+\frac{\beta_i \langle w'_{pi} v'_{pi}\rangle\, \partial \langle \omega'_{\varphi i} v'_{pi}\rangle}{r\psi_{5i}\partial r}
$$

$$
-\beta_i\left(\frac{1}{\tau_i}+\gamma_i\right)\langle \omega'_{\varphi i} w'_{pi}\rangle+\lambda_{\omega i}\beta_i (u_g-u_{pi})\langle \omega'_{\varphi i}\omega'_{ri}\rangle\ , \tag{23}
$$

$$
\beta_i u_{pi}\frac{\partial \langle u'_{pi}\omega'_{\varphi i}\rangle}{\partial z}+(\beta_i v_{pi}+\langle \beta'_i v'_{pi}\rangle)\frac{\partial \langle u'_{pi}\omega'_{\varphi i}\rangle}{\partial r}=\frac{\partial}{r\partial r}\left(\frac{r\beta_i \langle v'^{2}_{pi}\rangle\, \partial \langle u'_{pi}\omega'_{\varphi i}\rangle}{\psi_{5i}\partial r}\right)
$$

$$+\frac{\partial}{r\partial r}\left(\frac{r\beta_i\,\langle\omega'_{\varphi i}v'_{\mathrm{p}i}\rangle\,\partial\,\langle u'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle}{\psi_{5i}\partial r}\right)+\frac{\partial}{r\partial r}\left(\frac{r\beta_i\,\langle u'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle\,\partial\,\langle\omega'_{\varphi i}v'_{\mathrm{p}i}\rangle}{\psi_{5i}\partial r}\right)-\frac{\beta_i\,\langle\omega'_{\varphi i}v'_{\mathrm{p}i}\rangle\,\partial u_{\mathrm{p}i}}{\partial r}$$

$$-\frac{\beta_i\,\langle u'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle\,\partial\,\omega_{\varphi i}}{\partial r}-\beta_i\left(\frac{1}{\tau_i}+\gamma_i\right)\langle u'_{\mathrm{p}i}\omega'_{\varphi i}\rangle+(G_{\mathrm{p}5i}-D_{5i})\,, \tag{24}$$

$$\beta_i u_{\mathrm{p}i}\frac{\partial\,\langle v'^2_{\mathrm{p}i}\rangle}{\partial z}+(\beta_i v_{\mathrm{p}i}+\langle\beta'_i v'_{\mathrm{p}i}\rangle)\,\frac{\partial\,\langle v'^2_{\mathrm{p}i}\rangle}{\partial r}=\frac{\partial}{r\partial r}\left(r\tau_i\beta_i\,\langle v'^2_{\mathrm{p}i}\rangle\,\frac{\partial\,\langle v'^2_{\mathrm{p}i}\rangle}{\partial r}\right)$$

$$-\frac{2\partial\,(\tau_i\beta_i\,\langle w'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle^2)}{r\partial r}-2\beta_i\,\langle v'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle\,\frac{\partial v_{\mathrm{p}i}}{\partial r}-\frac{2\tau_i\beta_i\,\langle v'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle}{3r}\,\frac{\partial\,\langle w'^2_{\mathrm{p}i}\rangle}{\partial r}$$

$$-\frac{4\tau_i\beta_i\,\langle w'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle}{3r}\,\frac{\partial\,\langle w'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle}{\partial r}+\frac{4\tau_i\beta_i\,\langle w'^2_{\mathrm{p}i}\rangle^2}{3r^2}-\frac{4\tau_i\beta_i\,\langle v'^2_{\mathrm{p}i}\rangle\,\langle w'^2_{\mathrm{p}i}\rangle}{3r^2}-\frac{4\tau_i\beta_i\,\langle w'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle^2}{3r^2}$$

$$+\frac{2\beta_i\,(\langle v'_{\mathrm{p}i}v'_{\mathrm{g}}\rangle-\langle v'^2_{\mathrm{p}i}\rangle)}{\tau_i}-2\lambda_{\omega i}\beta_i\,(u_{\mathrm{g}}-u_{\mathrm{p}i})\,\langle\omega'_{\varphi i}v'_{\mathrm{p}i}\rangle+2\,(G_{\mathrm{p}6i}-D_{6i})\,, \tag{25}$$

$$\beta_i u_{\mathrm{p}i}\frac{\partial\,\langle w'^2_{\mathrm{p}i}\rangle}{\partial z}+(\beta_i v_{\mathrm{p}i}+\langle\beta'_i v'_{\mathrm{p}i}\rangle)\,\frac{\partial\,\langle w'^2_{\mathrm{p}i}\rangle}{\partial r}=\frac{\partial}{3r\partial r}\left(r\tau_i\beta_i\,\langle v'^2_{\mathrm{p}i}\rangle\,\frac{\partial\,\langle w'^2_{\mathrm{p}i}\rangle}{\partial r}\right)$$

$$+\frac{2\partial}{3r\partial r}\left(r\tau_i\beta_i\,\langle w'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle\,\frac{\partial\,\langle w'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle}{\partial r}\right)-\frac{2\partial\,(\tau_i\beta_i\,\langle w'^2_{\mathrm{p}i}\rangle^2)}{3r\partial r}+\frac{2\partial\,(\tau_i\beta_i\,\langle v'^2_{\mathrm{p}i}\rangle\,\langle w'^2_{\mathrm{p}i}\rangle)}{3r\partial r}$$

$$+\frac{2\partial\,(\tau_i\beta_i\,\langle w'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle^2)}{3r\partial r}-\frac{2\beta_i v_{\mathrm{p}i}\,\langle w'^2_{\mathrm{p}i}\rangle}{r}+\frac{2\tau_i\beta_i\,\langle v'^2_{\mathrm{p}i}\rangle\,\partial\,\langle w'^2_{\mathrm{p}i}\rangle}{3r\partial r}$$

$$+\frac{4\tau_i\beta_i\,\langle w'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle\,\partial\,\langle w'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle}{3r\partial r}-\frac{4\tau_i\beta_i\,\langle w'^2_{\mathrm{p}i}\rangle^2}{3r^2}+\frac{4\tau_i\beta_i\,\langle v'^2_{\mathrm{p}i}\rangle\,\langle w'^2_{\mathrm{p}i}\rangle}{3r^2}+\frac{4\tau_i\beta_i\,\langle w'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle^2}{3r^2}$$

$$+\frac{2\beta_i}{\tau_i}\,(\langle w'_{\mathrm{p}i}w'_{\mathrm{g}}\rangle-\langle w'^2_{\mathrm{p}i}\rangle)+2\lambda_{\omega i}\beta_i\,(u_{\mathrm{g}}-u_{\mathrm{p}i})\,\langle\omega'_{ri}w'_{\mathrm{p}i}\rangle+2\,(G_{\mathrm{p}7i}-D_{7i})\,, \tag{26}$$

$$\beta_i u_{\mathrm{p}i}\frac{\partial\,\langle w'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle}{\partial z}+(\beta_i v_{\mathrm{p}i}+\langle\beta'_i v'_{\mathrm{p}i}\rangle)\,\frac{\partial\,\langle w'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle}{\partial r}=\frac{2\partial}{3r\partial r}\left(r\tau_i\beta_i\,\langle v'^2_{\mathrm{p}i}\rangle\,\frac{\partial\,\langle w'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle}{\partial r}\right)$$

$$-\beta_i\,\langle w'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle\,\frac{\partial v_{\mathrm{p}i}}{\partial r}+\frac{2\partial\,(\tau_i\beta_i\,\langle v'^2_{\mathrm{p}i}\rangle\,\langle w'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle)}{3r\partial r}+\frac{\partial}{3r\partial r}\left(r\tau_i\beta_i\,\langle w'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle\,\frac{\partial\,\langle v'^2_{\mathrm{p}i}\rangle}{\partial r}\right)$$

$$-\frac{4\partial\left(\tau_i\beta_i\left\langle w'_{\mathrm{p}i}v'_{\mathrm{p}i}\right\rangle\left\langle w'^2_{\mathrm{p}i}\right\rangle\right)}{3r\partial r}-\frac{\tau_i\beta_i\left\langle w'_{\mathrm{p}i}v'_{\mathrm{p}i}\right\rangle}{r}\frac{\partial\left\langle w'^2_{\mathrm{p}i}\right\rangle}{\partial r}-\frac{10\tau_i\beta_i\left\langle w'_{\mathrm{p}i}v'_{\mathrm{p}i}\right\rangle\left\langle w'^2_{\mathrm{p}i}\right\rangle}{3r^2}$$

$$-\frac{\beta_i v_{\mathrm{p}i}\left\langle w'_{\mathrm{p}i}v'_{\mathrm{p}i}\right\rangle}{r}+\frac{2\tau_i\beta_i\left\langle v'^2_{\mathrm{p}i}\right\rangle}{3r}\frac{\partial\left\langle w'_{\mathrm{p}i}v'_{\mathrm{p}i}\right\rangle}{\partial r}+\frac{2\tau_i\beta_i\left\langle v'^2_{\mathrm{p}i}\right\rangle\left\langle w'_{\mathrm{p}i}v'_{\mathrm{p}i}\right\rangle}{3r^2}$$

$$+\frac{\tau_i\beta_i\left\langle w'_{\mathrm{p}i}v'_{\mathrm{p}i}\right\rangle}{3r}\frac{\partial\left\langle v'^2_{\mathrm{p}i}\right\rangle}{\partial r}+\frac{\beta_i}{\tau_i}\left(\left\langle v'_{\mathrm{g}}w'_{\mathrm{p}i}\right\rangle+\left\langle v'_{\mathrm{p}i}w'_{\mathrm{g}}\right\rangle-2\left\langle w'_{\mathrm{p}i}v'_{\mathrm{p}i}\right\rangle\right)$$

$$+\lambda_{\omega i}\beta_i\left(u_{\mathrm{g}}-u_{\mathrm{p}i}\right)\left(\left\langle\omega'_{ri}v'_{\mathrm{p}i}\right\rangle-\left\langle\omega'_{\varphi i}w'_{\mathrm{p}i}\right\rangle\right), \quad (27)$$

$$\beta_i u_{\mathrm{p}i}\frac{\partial\left\langle u'^2_{\mathrm{p}i}\right\rangle}{\partial z}+\left(\beta_i v_{\mathrm{p}i}+\left\langle\beta'_i v'_{\mathrm{p}i}\right\rangle\right)\frac{\partial\left\langle u'^2_{\mathrm{p}i}\right\rangle}{\partial r}=\frac{\partial}{3r\partial r}\left(r\tau_i\beta_i\left\langle v'^2_{\mathrm{p}i}\right\rangle\frac{\partial\left\langle u'^2_{\mathrm{p}i}\right\rangle}{\partial r}\right)$$

$$+2\tau_i\beta_i\left\langle v'^2_{\mathrm{p}i}\right\rangle\left(\frac{\partial u_{\mathrm{p}i}}{\partial r}\right)^2+\frac{2\beta_i}{\tau_i}\left(\left\langle u'_{\mathrm{p}i}u'_{\mathrm{g}}\right\rangle-\left\langle u'^2_{\mathrm{p}i}\right\rangle\right)+2\lambda_{\omega i}\beta_i\left(v_{\mathrm{g}}-v_{\mathrm{p}i}\right)\left\langle u'_{\mathrm{p}i}\omega'_{\varphi i}\right\rangle+2\left(G_{\mathrm{p}8i}-D_{8i}\right), \quad (28)$$

$$\beta_i u_{\mathrm{p}i}\frac{\partial\left\langle t'_{\mathrm{p}i}v'_{\mathrm{p}i}\right\rangle}{\partial z}+\left(\beta_i v_{\mathrm{p}i}+\left\langle\beta'_i v'_{\mathrm{p}i}\right\rangle\right)\frac{\partial\left\langle t'_{\mathrm{p}i}v'_{\mathrm{p}i}\right\rangle}{\partial r}=\frac{\partial}{r\partial r}\left(\frac{r\beta_i\left\langle v'^2_{\mathrm{p}i}\right\rangle}{\psi_{6i}}\frac{\partial\left\langle t'_{\mathrm{p}i}v'_{\mathrm{p}i}\right\rangle}{\partial r}\right)$$

$$+\frac{\partial}{2r\partial r}\left(\frac{r\beta_i\left\langle t'_{\mathrm{p}i}v'_{\mathrm{p}i}\right\rangle}{\psi_{6i}}\frac{\partial\left\langle v'^2_{\mathrm{p}i}\right\rangle}{\partial r}\right)-\frac{2\partial}{r\partial r}\left(\frac{\beta_i\left\langle w'_{\mathrm{p}i}v'_{\mathrm{p}i}\right\rangle\left\langle t'_{\mathrm{p}i}w'_{\mathrm{p}i}\right\rangle}{\psi_{6i}}\right)-\frac{\beta_i\left\langle v'^2_{\mathrm{p}i}\right\rangle\partial t_{\mathrm{p}i}}{\partial r}$$

$$-\frac{\beta_i\left\langle t'_{\mathrm{p}i}v'_{\mathrm{p}i}\right\rangle\partial v_{\mathrm{p}i}}{\partial r}-\frac{\beta_i\left\langle w'_{\mathrm{p}i}v'_{\mathrm{p}i}\right\rangle\partial\left\langle t'_{\mathrm{p}i}w'_{\mathrm{p}i}\right\rangle}{\psi_{6i}r\partial r}-\frac{\beta_i\left\langle t'_{\mathrm{p}i}v'_{\mathrm{p}i}\right\rangle\partial\left\langle w'^2_{\mathrm{p}i}\right\rangle}{2\psi_{6i}r\partial r}$$

$$-\frac{\beta_i\left\langle w'_{\mathrm{p}i}v'_{\mathrm{p}i}\right\rangle\left\langle t'_{\mathrm{p}i}w'_{\mathrm{p}i}\right\rangle}{\psi_{6i}r^2}-\frac{\beta_i\left\langle t'_{\mathrm{p}i}v'_{\mathrm{p}i}\right\rangle\left\langle w'^2_{\mathrm{p}i}\right\rangle}{\psi_{6i}r^2}+\frac{\beta_i\left(\left\langle t'_{\mathrm{p}i}v'_{\mathrm{g}}\right\rangle-\left\langle t'_{\mathrm{p}i}v'_{\mathrm{p}i}\right\rangle\right)}{\tau_i}$$

$$+\frac{6\beta_i\alpha_{\Sigma i}\left(\left\langle t'_{\mathrm{g}}v'_{\mathrm{p}i}\right\rangle-\left\langle t'_{\mathrm{p}i}v'_{\mathrm{p}i}\right\rangle\right)}{\rho_{\mathrm{p}i}c_{\mathrm{p}i}\delta_i}-\lambda_{\omega i}\beta_i\left(u_{\mathrm{g}}-u_{\mathrm{p}i}\right)\left\langle t'_{\mathrm{p}i}\omega'_{\varphi i}\right\rangle, \quad (29)$$

$$\beta_i u_{\mathrm{p}i}\frac{\partial\left\langle t'_{\mathrm{p}i}w'_{\mathrm{p}i}\right\rangle}{\partial z}+\left(\beta_i v_{\mathrm{p}i}+\left\langle\beta'_i v'_{\mathrm{p}i}\right\rangle\right)\frac{\partial\left\langle t'_{\mathrm{p}i}w'_{\mathrm{p}i}\right\rangle}{\partial r}=\frac{\partial}{r\partial r}\left(\frac{r\beta_i\left\langle v'^2_{\mathrm{p}i}\right\rangle}{\psi_{7i}}\frac{\partial\left\langle t'_{\mathrm{p}i}w'_{\mathrm{p}i}\right\rangle}{\partial r}\right)$$

$$+\frac{\partial}{r\partial r}\left(\frac{r\beta_i\left\langle t'_{\mathrm{p}i}v'_{\mathrm{p}i}\right\rangle}{\psi_{7i}}\frac{\partial\left\langle w'_{\mathrm{p}i}v'_{\mathrm{p}i}\right\rangle}{\partial r}\right)+\frac{\partial}{r\partial r}\left(\frac{\beta_i\left\langle t'_{\mathrm{p}i}v'_{\mathrm{p}i}\right\rangle\left\langle w'_{\mathrm{p}i}v'_{\mathrm{p}i}\right\rangle}{\psi_{7i}}\right)$$

$$
+\frac{\partial}{r\partial r}\left(\frac{r\beta_i\,\langle w'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle}{\psi_{7i}}\,\frac{\partial\,\langle t'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle}{\partial r}\right)-\frac{2\partial}{r\partial r}\left(\frac{\beta_i\,\langle t'_{\mathrm{p}i}w'_{\mathrm{p}i}\rangle\,\langle w'^2_{\mathrm{p}i}\rangle}{\psi_{7i}}\right)
$$

$$
+\frac{\partial}{r\partial r}\left(\frac{\beta_i\,\langle v'^2_{\mathrm{p}i}\rangle\,\langle t'_{\mathrm{p}i}w'_{\mathrm{p}i}\rangle}{\psi_{7i}}\right)-\frac{\beta_i\,\langle w'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle\,\partial\,t_{\mathrm{p}i}}{\partial r}-\frac{\beta_i v_{\mathrm{p}i}\,\langle t'_{\mathrm{p}i}w'_{\mathrm{p}i}\rangle}{r}+\frac{\beta_i\,\langle v'^2_{\mathrm{p}i}\rangle}{\psi_{7i}}\,\frac{\partial\,\langle t'_{\mathrm{p}i}w'_{\mathrm{p}i}\rangle}{r\partial r}
$$

$$
+\frac{\beta_i\,\langle t'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle}{\psi_{7i}}\,\frac{\partial\,\langle w'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle}{r\partial r}+\frac{\beta_i\,\langle t'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle\,\langle w'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle}{\psi_{7i}r^2}+\frac{\beta_i\,\langle w'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle}{\psi_{7i}}\,\frac{\partial\,\langle t'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle}{r\partial r}
$$

$$
-\frac{2\beta_i\,\langle w'^2_{\mathrm{p}i}\rangle\,\langle t'_{\mathrm{p}i}w'_{\mathrm{p}i}\rangle}{\psi_{7i}r^2}+\frac{\beta_i\,\langle v'^2_{\mathrm{p}i}\rangle\,\langle t'_{\mathrm{p}i}w'_{\mathrm{p}i}\rangle}{\psi_{7i}r^2}+\frac{\beta_i\,(\langle t'_{\mathrm{p}i}w'_{\mathrm{g}}\rangle-\langle t'_{\mathrm{p}i}w'_{\mathrm{p}i}\rangle)}{\tau_i}
$$

$$
+\frac{6\beta_i\alpha_{\Sigma i}\,(\langle t'_{\mathrm{g}}w'_{\mathrm{p}i}\rangle-\langle t'_{\mathrm{p}i}w'_{\mathrm{p}i}\rangle)}{\rho_{\mathrm{p}i}c_{\mathrm{p}i}\delta_i}+\lambda_{\omega i}\beta_i\,(u_{\mathrm{g}}-u_{\mathrm{p}i})\,\langle t'_{\mathrm{p}i}\omega'_{ri}\rangle\,, \tag{30}
$$

$$
\beta_i u_{\mathrm{p}i}\,\frac{\partial\,\langle t'_{\mathrm{p}i}\omega'_{\varphi i}\rangle}{\partial z}+(\beta_i v_{\mathrm{p}i}+\langle\beta'_i v'_{\mathrm{p}i}\rangle)\,\frac{\partial\,\langle t'_{\mathrm{p}i}\omega'_{\varphi i}\rangle}{\partial r}=\frac{\partial}{r\partial r}\left(\frac{r\beta_i\,\langle v'^2_{\mathrm{p}i}\rangle}{\psi_{8i}}\,\frac{\partial\,\langle t'_{\mathrm{p}i}\omega'_{\varphi i}\rangle}{\partial r}\right)
$$

$$
+\frac{\partial}{r\partial r}\left(\frac{r\beta_i\,\langle t'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle}{\psi_{8i}}\,\frac{\partial\,\langle\omega'_{\varphi i}v'_{\mathrm{p}i}\rangle}{\partial r}\right)+\frac{\partial}{r\partial r}\left(\frac{r\beta_i\,\langle\omega'_{\varphi i}v'_{\mathrm{p}i}\rangle}{\psi_{8i}}\,\frac{\partial\,\langle t'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle}{\partial r}\right)
$$

$$
-\beta_i\,\langle\omega'_{\varphi i}v'_{\mathrm{p}i}\rangle\,\frac{\partial t_{\mathrm{p}i}}{\partial r}-\beta_i\,\langle t'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle\,\frac{\partial\omega_{\varphi i}}{\partial r}+\frac{\beta_i\,\langle w'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle}{\psi_{8i}r}\,\frac{\partial\,\langle t'_{\mathrm{p}i}\omega'_{ri}\rangle}{\partial r}+\frac{\beta_i\,\langle\omega'_{ri}v'_{\mathrm{p}i}\rangle}{\psi_{8i}r}\,\frac{\partial\,\langle t'_{\mathrm{p}i}w'_{\mathrm{p}i}\rangle}{\partial r}
$$

$$
+\frac{\beta_i\,\langle t'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle}{\psi_{8i}r}\,\frac{\partial\,\langle\omega'_{ri}w'_{\mathrm{p}i}\rangle}{\partial r}-\frac{6\beta_i\alpha_{\Sigma i}\,\langle t'_{\mathrm{p}i}\omega'_{\varphi i}\rangle}{\rho_{\mathrm{p}i}c_{\mathrm{p}i}\delta_i}-\gamma_i\beta_i\,\langle t'_{\mathrm{p}i}\omega'_{\varphi i}\rangle\,, \tag{31}
$$

$$
\beta_i u_{\mathrm{p}i}\,\frac{\partial\,\langle t'_{\mathrm{p}i}\omega'_{ri}\rangle}{\partial z}+(\beta_i v_{\mathrm{p}i}+\langle\beta'_i v'_{\mathrm{p}i}\rangle)\,\frac{\partial\,\langle t'_{\mathrm{p}i}\omega'_{ri}\rangle}{\partial r}=\frac{\partial}{r\partial r}\left(\frac{r\beta_i\,\langle v'^2_{\mathrm{p}i}\rangle}{\psi_{8i}}\,\frac{\partial\,\langle t'_{\mathrm{p}i}\omega'_{ri}\rangle}{\partial r}\right)
$$

$$
+\frac{\partial}{r\partial r}\left(\frac{r\beta_i\,\langle\omega'_{ri}v'_{\mathrm{p}i}\rangle}{\psi_{8i}}\,\frac{\partial\,\langle t'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle}{\partial r}\right)+\frac{\partial}{r\partial r}\left(\frac{r\beta_i\,\langle t'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle}{\psi_{8i}}\,\frac{\partial\,\langle\omega'_{ri}v'_{\mathrm{p}i}\rangle}{\partial r}\right)-\beta_i\,\langle\omega'_{ri}v'_{\mathrm{p}i}\rangle\,\frac{\partial t_{\mathrm{p}i}}{\partial r}
$$

$$
+\frac{\beta_i\omega_{\varphi i}\,\langle t'_{\mathrm{p}i}w'_{\mathrm{p}i}\rangle}{r}-\frac{\beta_i\,\langle w'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle}{\psi_{8i}r}\,\frac{\partial\,\langle t'_{\mathrm{p}i}\omega'_{\varphi i}\rangle}{\partial r}-\frac{\beta_i\,\langle\omega'_{\varphi i}v'_{\mathrm{p}i}\rangle}{\psi_{8i}r}\,\frac{\partial\,\langle t'_{\mathrm{p}i}w'_{\mathrm{p}i}\rangle}{\partial r}
$$

$$
-\frac{\beta_i\,\langle t'_{\mathrm{p}i}v'_{\mathrm{p}i}\rangle}{\psi_{8i}r}\,\frac{\partial\,\langle\omega'_{\varphi i}w'_{\mathrm{p}i}\rangle}{\partial r}-\frac{6\beta_i\alpha_{\Sigma i}\,\langle t'_{\mathrm{p}i}\omega'_{ri}\rangle}{\rho_{\mathrm{p}i}c_{\mathrm{p}i}\delta_i}-\gamma_i\beta_i\,\langle t'_{\mathrm{p}i}\omega'_{ri}\rangle\,, \tag{32}
$$

where we have

$$\psi_{1i} = \frac{1}{\tau_i} + \gamma_i\,; \quad \psi_{2i} = \frac{1}{\tau_i} + \frac{\gamma_i}{2}\,; \quad \psi_{3i} = \frac{1}{2\tau_i} + \gamma_i\,; \quad \psi_{4i} = \frac{1}{\tau_i} + 2\gamma_i\,; \quad \psi_{5i} = \frac{2}{\tau_i} + \gamma_i\,;$$

$$\psi_{6i} = \frac{3\alpha_{\Sigma i}}{\rho_{pi} c_{pi} \delta_i} + \frac{1}{\tau_i}\,; \quad \psi_{7i} = \frac{6\alpha_{\Sigma i}}{\rho_{pi} c_{pi} \delta_i} + \frac{2}{\tau_i}\,; \quad \psi_{8i} = \gamma_i + \frac{1}{\tau_i} + \frac{6\alpha_{\Sigma i}}{\rho_{pi} c_{pi} \delta_i}\,.$$

The mixed moments $\langle v'_{pi} v'_g \rangle$, $\langle w'_{pi} w'_g \rangle$, $\langle v'_g w'_{pi} \rangle$, $\langle v'_{pi} w'_g \rangle$, $\langle u'_{pi} u'_g \rangle$, $\langle t'_g v'_{pi} \rangle$, $\langle t'_{pi} v'_g \rangle$, $\langle t'_{pi} w'_g \rangle$, and $\langle t'_g w'_{pi} \rangle$ which are present in Eqs. (25)–(30) are determined in terms of the correlations of the gas in a locally homogeneous approximation according to the recommendations of [6]. The system of equations (17)–(32) is open, since the extra terms of nonturbulent origin that describe the pseudoturbulence generation G_{pqi} and dissipation D_{qi} caused by the interaction of mono- and polydisperse particles due to their random motion appear in it. To determine G_{pqi} and D_{qi} we use a special computational procedure that is based on an analysis of the dynamics of the process of collisions [2, 4, 7].

Generation of Pseudoturbulence. The rate of creation of the second moments $\langle \omega'_{ri} w'_{pi} \rangle$ and $\langle \omega'_{\varphi i} v'_{pi} \rangle$ is

$$G_{pqi} = \frac{\beta_i\,(1 - K_\tau)}{21\delta_i} \left\{ \frac{(1 - K_\tau)\, s_{qi} \delta_i^2 \omega_{\varphi i}^2}{14} - \frac{a_{qi} \left[\frac{1 - K_n}{2} - \frac{1 - K_\tau}{7} \right] \delta_m^2}{144\beta_m^2} \left(\frac{\partial u_{pi}}{\partial r} \right)^2 \right\} N_{\Sigma i,i} + \frac{4\,(1 - K_\tau)}{21}$$

$$\times \sum_{\substack{i=1 \\ y \neq i}}^{M_C + M_{ash}} \frac{\beta_i m_y^2 N_{\Sigma y,i}}{(m_y + m_i)^2\, \delta_i} \left\{ \frac{(1 - K_\tau)\, (\delta_i \omega_{\varphi i} + \delta_y \omega_{\varphi y})^2\, s_{qi}}{56} - a_{qi} \left[1 - K_n - \frac{2\,(1 - K_\tau)}{7} \right] (u_{py} - u_{pi})^2 \right\},$$

the rate of generation of the components of the pulsation energy of rotational motion of particles $\langle \omega'^2_{ri} \rangle$ and $\langle \omega'^2_{\varphi i} \rangle$ is

$$G_{pqi} = \frac{\beta_i\,(1 - K_\tau)^2}{s_{qi} \delta_i^2} \left[\frac{\delta_m^2}{a_{qi} \beta_m^2} \left(\frac{\partial u_{pi}}{\partial r} \right)^2 + \delta_i^2 \omega_{\varphi i}^2 \right] N_{\Sigma i,i} + \sum_{\substack{i=1 \\ y \neq i}}^{M_C + M_{ash}} \frac{(1 - K_\tau)^2\, m_y^2 \beta_i N_{\Sigma y,i}}{3.92\, (m_y + m_i)^2\, \delta_i^2}$$

$$\times \left[(u_{py} - u_{pi})^2 + \frac{(\delta_i \omega_{\varphi i} + \delta_y \omega_{\varphi y})^2}{f_{qi}} \right],$$

the rate of production of the correlation of second order $\langle u'_{pi} \omega'_{\varphi i} \rangle$ is

$$G_{p5i} = -\frac{(1 - K_\tau)\, \omega_{\varphi i} \delta_m}{806.4\sqrt{2}\, \beta_m} \frac{\partial u_{pi}}{\partial r} \left[1 - K_n + \frac{5\,(1 - K_\tau)}{3.5} \right] \beta_i N_{\Sigma i,i}$$

$$- \frac{1 - K_\tau}{1.4} \sum_{\substack{i=1 \\ y \neq i}}^{M_C + M_{ash}} \frac{m_y^2 \beta_i N_{\Sigma y,i}\, (u_{py} - u_{pi})\, (\delta_i \omega_{\varphi i} + \delta_y \omega_{\varphi y})}{\delta_i\, (m_y + m_i)^2} \left[\frac{1 - K_n}{24} + \frac{5\,(1 - K_\tau)}{84} \right],$$

the rate of creation of the components of the pulsation energy of translational motion of particles $\langle v'^2_{pi} \rangle$, $\langle w'^2_{pi} \rangle$, and $\langle u'^2_{pi} \rangle$ is

$$G_{pqi} = \left\{ \frac{\left[\frac{1-K_n}{2} - \frac{1-K_\tau}{7}\right]^2 \delta_m^2}{6912\beta_m^2} \left(\frac{\partial u_{pi}}{\partial r}\right)^2 + \frac{s_{qi}\delta_i^2 (1-K_\tau)^2 \omega_{\varphi i}^2}{a_{qi}} \right\} \beta_i N_{\Sigma i,i} + \sum_{\substack{i=1 \\ y\neq i}}^{M_C+M_{ash}} \frac{m_y^2 \beta_i N_{\Sigma y,i}}{(m_y+m_i)^2}$$

$$\times \left\{ \frac{\left[1-K_n - \frac{2(1-K_\tau)}{7}\right]^2 (u_{py}-u_{pi})^2}{24} + \frac{s_{qi}(1-K_\tau)^2 (\delta_i\omega_{\varphi i} + \delta_y \omega_{\varphi y})^2}{a_{qi}} \right\}, \qquad (33)$$

where $q = 1$ and 2 refer to $\langle \omega'_{ri} w'_{pi}\rangle$, $\langle \omega'_{\varphi i} v'_{pi}\rangle$, $s_{1i} = 0$, $s_{2i} = 1$, $a_{1i} = -1$, $a_{2i} = 1$, and $K_n < 0$; $q = 3$ and 4 refer to $\langle \omega'^2_{ri}\rangle$, $\langle \omega'^2_{\varphi i}\rangle$, $s_{3i} = 94.08$, $s_{4i} = 62.72$, $a_{3i} = 48$, $a_{4i} = 72$, $f_{3i} = 24$, and $f_{4i} = 16$; and $q = 6$, 7, and 8 refer to $\langle v'^2_{pi}\rangle$, $\langle w'^2_{pi}\rangle$, and $\langle u'^2_{pi}\rangle$, $s_{6i} = 1$, $s_{7i} = 0$; $s_{8i} = 1$; $a_{6i} = 1764$, $a_{7i} = 0$, and $a_{8i} = 392$.

Dissipation of Pseudoturbulence. The rate of dissipation of the correlations $\langle \omega'_{ri} w'_{pi}\rangle$ and $\langle \omega'_{\varphi i} v'_{pi}\rangle$ is

$$D_{qi} = U_q \sum_{i=1}^{M_C+M_{ash}} \beta_i N_{\Sigma y,i} \left\{ \frac{2(1-K_\tau)^2 \langle \boldsymbol{\omega}'^2_{\Sigma y,i}\rangle m_y^2 A_{qi}}{73.5\delta_i (m_y+m_i)^2} - \frac{X_{qi}(1-K_\tau)\sqrt{\langle \boldsymbol{\Omega}'^2_i\rangle \langle \boldsymbol{\omega}'^2_{\Sigma y,i}\rangle} \cos\vartheta_{y,i} m_y}{21(m_y+m_i)} \right.$$

$$\left. - \frac{W_{qi}(1-K_\tau)\left[1-K_n - \frac{2(1-K_\tau)}{7}\right] m_y^2 \langle \mathbf{V}'^2_{py,i}\rangle}{5.25\delta_i (m_y+m_i)^2} \right\},$$

the rate of dissipation of the second moments $\langle \omega'^2_{ri}\rangle$ and $\langle \omega'^2_{\varphi i}\rangle$ is

$$D_{qi} = U_q \sum_{i=1}^{M_C+M_{ash}} \beta_i N_{\Sigma y,i} \left\{ \frac{3(1-K_\tau)\sqrt{\langle \boldsymbol{\Omega}'^2_i\rangle \langle \boldsymbol{\omega}'^2_{\Sigma y,i}\rangle} \cos\vartheta_{y,i} m_y}{5.6\delta_i (m_y+m_i)} - \frac{(1-K_\tau)^2 m_y^2}{1.96(m_y+m_i)^2 \delta_i^2} \times \left[\frac{\langle \boldsymbol{\omega}'^2_{\Sigma y,i}\rangle}{3} + \langle \mathbf{V}'^2_{py,i}\rangle\right] \right\},$$

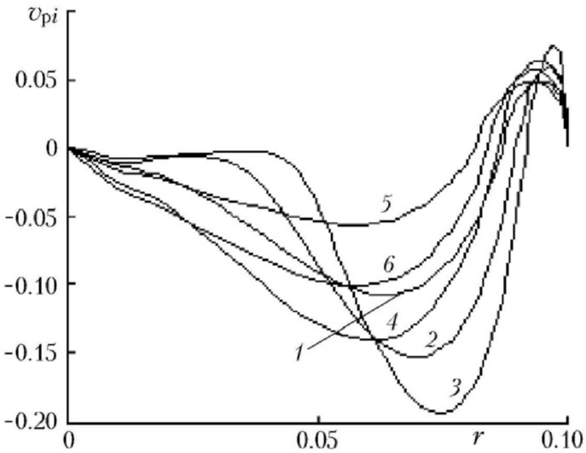
the rate of dissipation of the components of the pulsation energy of translational motion of particles $\langle v'^2_{pi}\rangle$, $\langle w'^2_{pi}\rangle$, and $\langle u'^2_{pi}\rangle$ is

$$D_{qi} = U_q \sum_{i=1}^{M_C+M_{ash}} \frac{m_y^2 \beta_i N_{\Sigma y,i}}{(m_y+m_i)^2} \left\{ \frac{\left[1 - \left(\frac{1-K_n}{2} - \frac{1-K_\tau}{7}\right)^2\right] \langle \mathbf{V}'^2_{py,i}\rangle}{3} - \frac{(1-K_\tau)^2 \langle \boldsymbol{\omega}'^2_{\Sigma y,i}\rangle}{196} \right\}, \qquad (34)$$

where $q = 1$ and 2 refer to $\langle \omega'_{ri} w'_{pi}\rangle$, $\langle \omega'_{\varphi i} v'_{pi}\rangle$, $A_{1i} = 1$, $A_{2i} = -1$, $X_{1i} = 1$, $X_{2i} = -1$, $W_{1i} = 1$, and $W_{2i} = -1$; $q = 3$ and 4 refer to $\langle \omega'^2_{ri}\rangle$, $\langle \omega'^2_{\varphi i}\rangle$, and $D_{5i} = 0$; and $q = 6$, 7, and 8 refer to $\langle v'^2_{pi}\rangle$, $\langle w'^2_{pi}\rangle$, and $\langle u'^2_{pi}\rangle$

$$\langle \boldsymbol{\Omega}'^2_i\rangle^{0.5} = \sqrt{\langle \omega'^2_{\varphi i}\rangle + \langle \omega'^2_{ri}\rangle}, \quad \langle \boldsymbol{\omega}'^2_{\Sigma y,i}\rangle^{0.5} = \frac{2}{\pi}\left(\delta_i \langle \boldsymbol{\Omega}'^2_i\rangle^{0.5} + \delta_y \langle \boldsymbol{\Omega}'^2_y\rangle^{0.5}\right) E_{y,i},$$

$$\cos\vartheta_{y,i} = \frac{\langle \boldsymbol{\omega}'^2_{\Sigma y,i}\rangle + \delta_i^2 \langle \boldsymbol{\Omega}'^2_i\rangle - \delta_y^2 \langle \boldsymbol{\Omega}'^2_y\rangle}{2\langle \boldsymbol{\omega}'^2_{\Sigma y,i}\rangle^{0.5} \delta_i \langle \boldsymbol{\Omega}'^2_i\rangle^{0.5}}.$$



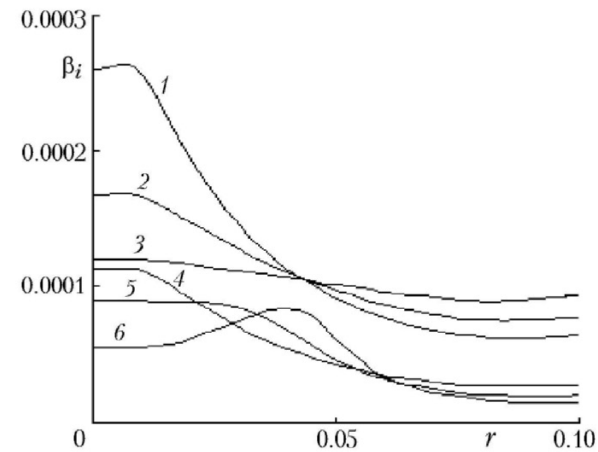
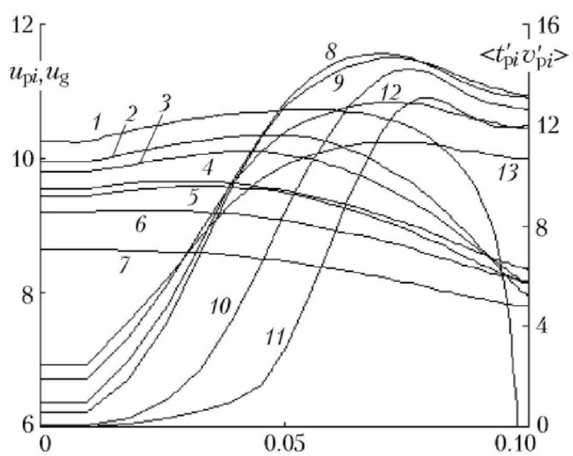


Fig. 1. Distribution of the averaged radial velocities of coke particles of diameter δ_{C0} (1) $0.3 \cdot 10^{-3}$ m, 2) $0.2 \cdot 10^{-3}$ m, and 3) $0.15 \cdot 10^{-3}$ m) and ash particles of diameter δ_{ash} (4) $0.2 \cdot 10^{-3}$ m, 5) $0.35 \cdot 10^{-3}$ m, and 6) $0.26 \cdot 10^{-3}$ m) over the reactor cross section at the mark z = 6 m.

Fig. 2. Distribution of the volume concentrations of particles over the reactor cross section at the mark z = 6 m: 1) β_{ash1}; 2) β_{ash3}; 3) β_{ash2}; 4) β_{C1}; 5) β_{C2}; 6) β_{C3}.



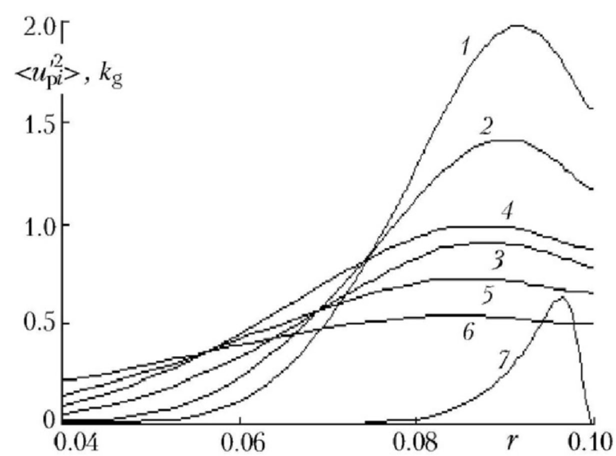
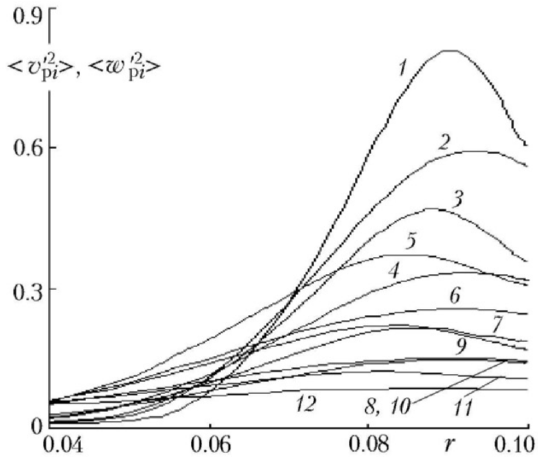


Fig. 3. Distribution of the averaged axial velocities of phases and the second moments of pulsations of the temperature and radial velocity of particles over the reactor cross section at the mark z = 6 m: 1) u_g; 2) u_{C3}; 3) u_{C2}; 4) u_{ash1}; 5) u_{C1}; 6) u_{ash3}; 7) u_{ash2}; 8) $\langle t'_{C1} v'_{C1} \rangle$; 9) $\langle t'_{ash1} v'_{ash1} \rangle$; 10) $\langle t'_{C2} v'_{C2} \rangle$; 11) $\langle t'_{C3} v'_{C3} \rangle$; 12) $\langle t'_{ash3} v'_{ash3} \rangle$; 13) $\langle t'_{ash2} v'_{ash2} \rangle$.

Fig. 4. Distribution of the axial component of the pulsation energy of coke particles of diameter δ_{C0} (1) $0.15 \cdot 10^{-3}$ m, 2) $0.2 \cdot 10^{-3}$ m, and 3) $0.3 \cdot 10^{-3}$ m) and ash particles of diameter δ_{ash} (4) $0.2 \cdot 10^{-3}$ m, 5) $0.26 \cdot 10^{-3}$ m, and 6) $0.35 \cdot 10^{-3}$ m) and of the turbulent energy of the gas (7) k_g) over the reactor cross section at the mark z = 6 m.

Here $y = 1 - M_C + M_{ash}$, $\langle \mathbf{V}'^2_{py,i} \rangle$ is the mean relative velocity squared of particles y and i in random motion [2], and $N_{\Sigma i,i}$ and $N_{\Sigma y,i}$ are the total frequencies of collisions of mono- and polydisperse particles, which includes collisions due to the difference of the averaged axial velocities of the particles [2, 7] and their random motion [2]. The boundary conditions on the axis (r = 0) and wall of the channel ($r = R$) for Eqs. (1)–(14) and (17)–(30) are formed similarly to [4, 5]; for Eqs. (31) and (32), they are formed according to the relations



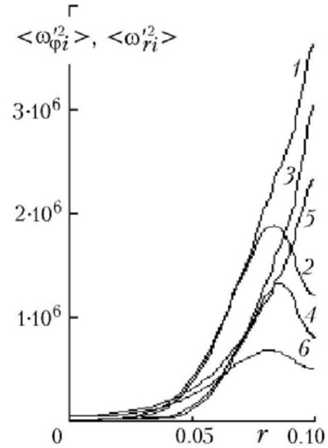
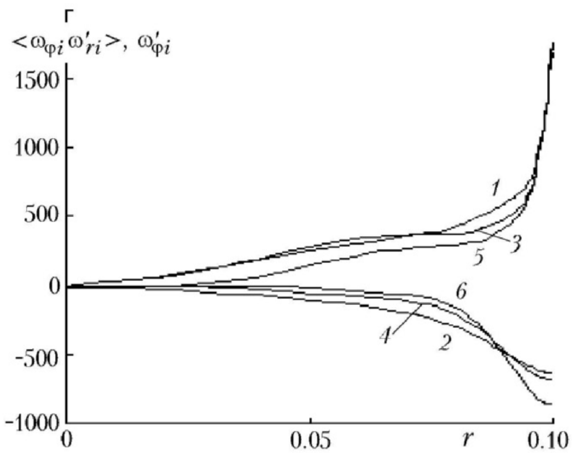


Fig. 5. Distribution of the second moments of pulsations of the translational velocity of particles over the reactor cross section at the mark z = 6 m: 1) $\langle w'_{C3}w'_{C3} \rangle$; 2) $\langle v'_{C3}v'_{C3} \rangle$; 3) $\langle w'_{C2}w'_{C2} \rangle$; 4) $\langle v'_{C2}v'_{C2} \rangle$; 5) $\langle w'_{ash1}w'_{ash1} \rangle$; 6) $\langle v'_{ash1}v'_{ash1} \rangle$; 7) $\langle w'_{ash3}w'_{ash3} \rangle$; 8) $\langle v'_{ash3}v'_{ash3} \rangle$; 9) $\langle w'_{C1}w'_{C1} \rangle$; 10) $\langle v'_{C1}v'_{C1} \rangle$; 11) $\langle w'_{ash2}w'_{ash2} \rangle$; 12) $\langle v'_{ash2}v'_{ash2} \rangle$.

Fig. 6. Distribution of the second moments of pulsations of the angular velocity of particles over the flow cross section at the mark z = 6 m: 1) $\langle \omega'^2_{\varphi ash3} \rangle$; 2) $\langle \omega'^2_{rash3} \rangle$; 3) $\langle \omega'^2_{\varphi C1} \rangle$; 4) $\langle \omega'^2_{rC1} \rangle$; 5) $\langle \omega'^2_{\varphi ash2} \rangle$; 6) $\langle \omega'^2_{rash2} \rangle$.



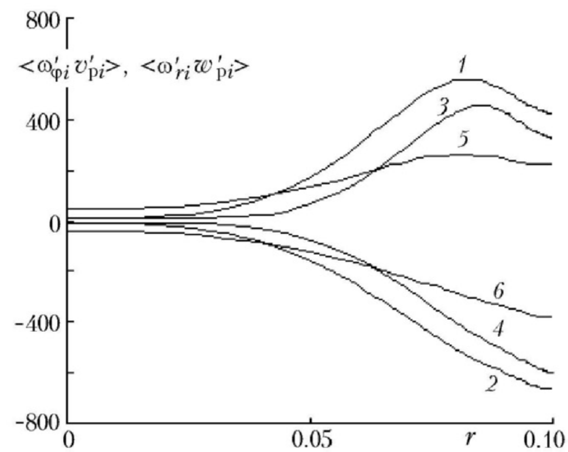
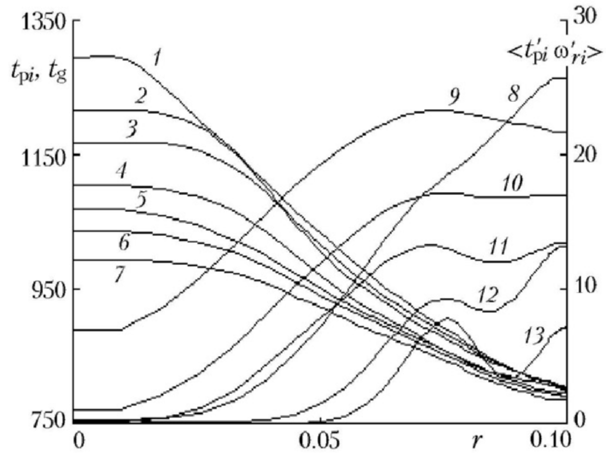


Fig. 7. Distribution of the second moments and the averaged velocities of particles over the flow cross section at the mark z = 6 m: 1) $\omega_{\varphi ash2}$; 2) $\langle \omega'_{\varphi ash2}\omega'_{rash2} \rangle$; 3) $\omega_{\varphi ash3}$; 4) $\langle \omega'_{\varphi ash3}\omega'_{rash3} \rangle$; 5) $\omega_{\varphi C1}$; 6) $\langle \omega'_{\varphi C1}\omega'_{rC1} \rangle$.

Fig. 8. Distribution of the second moments of pulsations of the translational and angular velocity of particles over the flow cross section at the mark z = 6 m: 1) $\langle \omega'_{rash3}w'_{ash3} \rangle$; 2) $\langle \omega'_{\varphi ash3}v'_{ash3} \rangle$; 3) $\langle \omega'_{rC1}w'_{C1} \rangle$; 4) $\langle \omega'_{\varphi C1}v'_{C1} \rangle$; 5) $\langle \omega'_{rash2}w'_{ash3} \rangle$; 6) $\langle \omega'_{\varphi ash2}v'_{ash2} \rangle$.

$$\left(\frac{\partial \langle t'_{pi}\omega'_{ri} \rangle}{\partial r}\right)_{r=0} = \left(\frac{\partial \langle t'_{pi}\omega'_{\varphi i} \rangle}{\partial r}\right)_{r=0} = 0 \,, \quad \left(\frac{\partial \langle t'_{pi}\omega'_{ri} \rangle}{\partial r}\right)_{r=R} = \left(\frac{\partial \langle t'_{pi}\omega'_{\varphi i} \rangle}{\partial r}\right)_{r=R} = 0 \,. \tag{35}$$

The complete system of equations (1)–(15) and (17)–(32) with account for expressions (16), (33), and (34) and with boundary conditions (35) contain equations of three types. The parabolic and hyperbolic equations are inte-



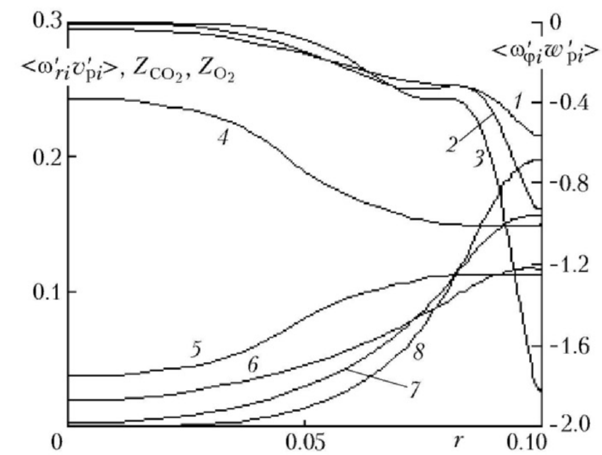


Fig. 9. Distribution of the averaged temperatures of phases and the second moments of pulsations of the temperature and angular velocity of particles over the reactor cross section at the mark z = 6 m: 1) t_{C1}; 2) t_{C2}; 3) t_{C3}; 4) t_g; 5) t_{ash1}; 6) t_{ash3}; 7) t_{ash2}; 8) $\langle t'_{C1}\omega'_{rC1}\rangle$; 9) $\langle t'_{ash2}\omega'_{rash2}\rangle$; 10) $\langle t'_{ash3}\omega'_{rash3}\rangle$; 11) $\langle t'_{ash1}\omega'_{rash1}\rangle$; 12) $\langle t'_{C2}\omega'_{rC2}\rangle$; 13) $\langle t'_{C3}\omega'_{rC3}\rangle$.

Fig. 10. Distribution of the components of the gaseous mixture and the second moments of pulsations of the translational and angular velocities of particles over the flow cross section at the mark z = 6 m: 1) $\langle \omega'_{\varphi ash2} w'_{ash2}\rangle$; 2) $\langle \omega'_{\varphi ash3} w'_{ash3}\rangle$; 3) $\langle \omega'_{\varphi C1} w'_{C1}\rangle$; 4) Z_{CO_2}; 5) Z_{O_2}; 6) $\langle \omega'_{rash2} v'_{ash2}\rangle$; 7) $\langle \omega'_{rash3} v'_{ash3}\rangle$; 8) $\langle \omega'_{rC1} v'_{C1}\rangle$.

grated by the method of direct and reverse runs on a uniform grid bunching at the channel wall. The iteration method is used for solution of the equations of second order; no iterations are required for the equations of first order. The equation of continuity of the carrying medium is approximated by an implicit four-point scheme. In accordance with the above algorithm, we have developed a program for calculation of nonisothermal two-phase flows.

Certain Calculation Results. We discuss results of calculations of the combustion of a polydisperse ensemble of coke-ash particles of anthracite culm in an axisymmetric channel for the following initial data: $Z_{O_2 0}$ = 0.12, R = 0.1 m, $\delta_{C01} = 0.3 \cdot 10^{-3}$ m, $\delta_{C02} = 0.2 \cdot 10^{-3}$ m, $\delta_{C03} = 0.15 \cdot 10^{-3}$ m, $\delta_{ash1} = 0.2 \cdot 10^{-3}$ m, $\delta_{ash2} = 0.35 \cdot 10^{-3}$ m, $\delta_{ash3} = 0.26 \cdot 10^{-3}$ m, $\beta_{C01} = \beta_{C02} = \beta_{C03} = 0.0001$, $\beta_{ash1} = \beta_{ash2} = \beta_{ash3} = 0.0002$, B_{g0} = 359 kg/h, ρ_C = 1000 kg/m^3, ρ_{ash} = 1800 kg/m^3, $t_{pi0} = t_{g0} = 900^{o}$C, $t_w = 800^{o}$C, and $u_{g,m0}$ = 10 m/sec.

Figure 1 illustrates the profiles of the radial velocities of the polydisperse ensemble of coke-ash particles. It is seen that v_{pi} is less than 0 in a large part of the channel cross section (except for the wall zone), which gives a maximum of the $\beta_i(r)$ function at the flow axis and its minimum in the peripheral zone (Fig. 2). Such behavior of the $\beta_i(r)$ curve leads to the fact that the aerodynamic-drag force in the axial zone $F_{aiz} \sim \beta_i(u_g - u_{pi})$ (see (16)) becomes larger than that in the peripheral zone. Therefore, the gas velocity diminishes at the axis and grows in the wall region (Fig. 3, curve 1). It follows from Fig. 3 that curves 1, 2, and 3 are similar, whereas plots 1 and 7 differ. The reason is that small coke particles, unlike large inertial ash particles, are well involved in the pulsatory motion of the carrying medium.

Figures 4 and 5 show the distributions of the components of the pulsation energy of translational motion of the dispersed phase and the turbulent energy of the carrying medium. It is seen that the energy of random motion of particles $0.5(\langle v'^2_{pi}\rangle + \langle w'^2_{pi}\rangle + \langle u'^2_{pi}\rangle)$ is much higher than k_g. This points to the dominant role of the pseudoturbulent mechanism of transfer in the mechanics of motion of highly concentrated polydisperse flows.

Figure 6 gives results of calculations of the second moments of pulsations of the angular particle velocity. An analysis of the balance of terms of Eq. (19) shows that the behavior of the function $\langle \omega'^2_{\varphi i}\rangle$ (r) (curves 1, 3, and 5) is mainly dependent on the rate of generation of the pulsation-energy component $\langle \omega'^2_{\varphi i}\rangle$ caused by the transition of the averaged rotational motion to pulsatory motion (third term of the right-hand side of Eq. (19)) and by the interparticle interaction (eighth term of the right-hand side of Eq. (19) G_{p4i} (see (33)). Growth in curves 1, 3, and 5 is related to

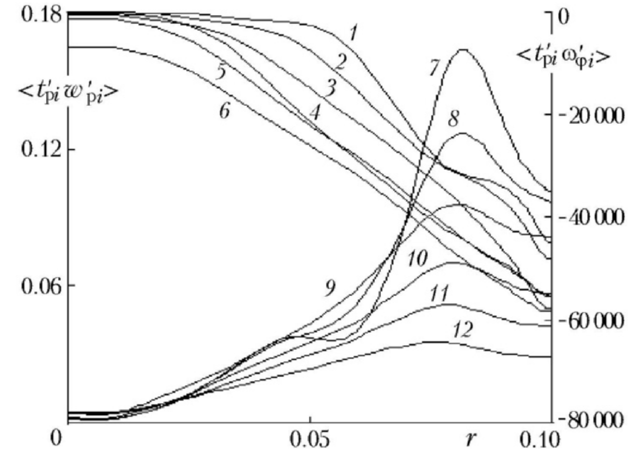


Fig. 11. Distribution of the second moments of pulsations of the temperature and translational and angular velocities of particles over the reactor cross section at the mark z = 6 m: 1) $\langle t'_{C3}\omega'_{\varphi C3}\rangle$; 2) $\langle t'_{C2}\omega'_{\varphi C2}\rangle$; 3) $\langle t'_{C1}\omega'_{\varphi C1}\rangle$; 4) $\langle t'_{ash1}\omega'_{\varphi ash1}\rangle$; 5) $\langle t'_{ash3}\omega'_{\varphi ash3}\rangle$; 6) $\langle t'_{ash2}\omega'_{\varphi ash2}\rangle$; 7) $\langle t'_{C3}w'_{C3}\rangle$; 8) $\langle t'_{C2}w'_{C2}\rangle$; 9) $\langle t'_{C1}w'_{C1}\rangle$; 10) $\langle t'_{ash1}w'_{ash1}\rangle$; 11) $\langle t'_{ash3}w'_{ash3}\rangle$; 12) $\langle t'_{ash2}w'_{ash2}\rangle$.

the rapid growth in the angular particle velocity $\omega_{\varphi i}$ and in the derivatives $\partial\omega_{\varphi i}/\partial r$ and $|\partial u_{pi}/\partial r|$ (Fig. 3, curves 5–7; Fig. 7, curves 1, 3, and 5) and to the decrease in the dependence $\langle\omega'_{\varphi i}v'_{pi}\rangle(r)$ in this zone (Fig. 8, curves 2, 4, and 6).

Figure 9 gives the profiles of the phase temperatures and the second moments of pulsations of the particle temperature and angular velocity. In the central part of the reactor where the carbon concentration is maximum (Fig. 2, curves 4–6), we have intense oxygen burning to form CO_2 (Fig. 10, curves 4 and 5), because of which the heat release from the coke particles exceeds (convective + radiant) heat removal; this ensures their efficient burning, whose intensity increases toward the flow axis. Therefore, the coke temperature is higher that the gas and ash-particle temperature (Fig. 9, curves 1–7).

Figure 11 illustrates the distribution of the second moments of pulsations of the temperature and transactional and angular velocities of particles over the cross section of the two-phase flow. It is seen that the $\langle t'_{pi}w'_{pi}\rangle(r)$ function (curve 8) has its maximum at the point r = 0.082 m. An analysis of the calculation results shows (see (30)) that the rapid growth in the $\langle t'_{pi}w'_{pi}\rangle(r)$ curve on the ascending branch 0.0095 m < r < 0.082 m is related to the increase in the functions $\langle w'_{pi}v'_{pi}\rangle(r)$, $\langle t'_{pi}w'_{g}\rangle(r)$, and $\langle t'_{g}w'_{pi}\rangle(r)$ and to the decrease in the dependence $t_{pi}(r)$ in this zone. Decrease in the function $\langle t'_{pi}w'_{pi}\rangle(r)$ on the descending portion 0.082 m < r is caused by the decrease in $\langle w'_{pi}v'_{pi}\rangle$, $\langle t'_{pi}w'_{g}\rangle$, and $\langle t'_{g}w'_{pi}\rangle$.

Conclusions. The proposed model makes it possible to correctly determine the aerodynamic and physicochemical characteristics of the ascending polydisperse coke-ash-particle flow. This gives grounds to use it in designing chemical reactors in the stages of engineering and detailed design.

NOTATION

A, X, W, a, s, and f, coefficients; B, flow rate, kg/sec; C, concentration, kmole/m^3; c, heat capacity, kJ/(kg·K); D, dissipation of pseudoturbulence, m^2/sec^3, m/sec^3, and sec^{-3}; E, complete elliptic integral; F, force, kg/(sec^2·m^2); G, generation of turbulent (pseudoturbulent) energy, kg/(sec^3·m), (m^2/sec^3, m/sec^3, and sec^{-3}); g, free-fall generation, m/sec^2; H, universal gas constant, kJ/(kmole·K); J, turbulent-diffusion coefficient, m^2/sec; K, coefficient of restitution; k, kinetic pulsation energy, m^2/sec^2; L, reaction-rate constant, m/sec; M, number of fractions; m, particle mass, kg; N, collision frequency, 1/sec; n, number concentration of particles; P, gas pressure, N/m^2; Pr, Prandtl number; Q, thermal effect of the reaction $C + O_2 = CO_2$, kJ/kmole; R, channel radius, m; r, z, radial and longitudinal coordinates, m; S, mass-exchange coefficient, m/sec; t, temperature, ^{o}C; u, v, w, averaged velocity-vector components, m/sec; U, empirical constant; **V**, vector of translational particle velocity; Z, weight fraction of the component of the gas mixture; α, coefficient of heat exchange between the gas and the particle, kJ/(sec·m^2·K); β, true volume concentration of particles; γ, coefficient, sec^{-1}; δ, particle diameter, m; ε, dissipation of pulsation energy, m^2/sec^3; η, kinematic viscosity, m^2/sec; θ and λ, coefficients; μ, molecular weight, kg/kmole; ρ, density, kg/m^3; σ, empirical constant; τ, dynamic-relaxation time, sec; ϑ, angle; ψ_1–ψ_8, coefficients, sec^{-1}; $\boldsymbol{\omega}$, sum of the vectors of angular particle velocities; $\boldsymbol{\Omega}$, vector of angular particle velocity. Subscripts and superscripts: a, aerodynamic drag of a particle; ash, ash; col, collisions; g, gas; m, mean value; M, Magnus force; n, normal; p, particle; q = 1–8: 1) $\langle \omega'_{ri} w'_{pi} \rangle$, 2) $\langle \omega'_{\varphi i} v'_{pi} \rangle$, 3) $\langle \omega'^2_{ri} \rangle$, 4) $\langle \omega'^2_{\varphi i} \rangle$, 5) $\langle u'_{pi} \omega'_{\varphi i} \rangle$, 6) $\langle v'^2_{pi} \rangle$, 7) $\langle w'^2_{pi} \rangle$, 8) $\langle u'^2_{pi} \rangle$; t, pulsations; w, channel wall; τ, tangential; φ, transverse coordinate; χ = 1–3 refer to O_2, CO_2, and N_2; ω, angular velocity of a particle; Σ, total (radiant + convective) heat exchange (or total frequency of collisions due to the random and averaged motion); 0, initial conditions; 1, 2, and 3, fraction numbers of coke and ash particles; $'$, pulsation component in time averaging; $\langle\rangle$, averaging over time (space).

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